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(दूसरा पुनरीक्षण)

Indian Standard

METHODS FOR STATISTICAL QUALITY CONTROL DURING PRODUCTION

PART 1 CONTROL CHARTS FOR VARIABLES

(Second Revision)

ICS 03.120.30

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BUREAU OF INDIAN STANDARDS
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NEW DELHI 110002

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FOREWORD

This Indian Standard (Part 1) (Second Revision) was adopted by the Bureau of Indian Standards, after the draft finalized by the Statistical Methods for Quality and Reliability Sectional Committee had been approved by the Management and Systems Division Council.

Controlling the quality of products so as to maintain it at a given level is a major problem with all the producers. From the early days of industrial production manufacturers have tried to use the same men, machines and methods and similar raw materials in the hope of turning out products of uniform quality. But neither men nor machines are infallible and causes of irregularity often creep inadvertently. As a result, rejections in finished materials are rarely eliminated and inspection and screening become necessary to varied extents determined by the nature of the product and the goodwill and policy of the manufacturer.

Since screening is not an effective control, the question remained a lively issue with the production management and various systems of control were devised from time-to-time. In 1924 Dr. W.A. Shewhart of the Bell Telephone Laboratories, USA developed the control chart method of controlling the quality during production which is meant to be an integral part of the production process. This technique based on statistical methods, however, does not provide an automatic corrective action in the way mechanical or electrical systems do. Instead, it gives a warning signal to the operator that he must take here and now the corrective action on his machine or process to ensure maintenance of quality in further production. Its effectiveness, therefore, depends on the promptness with which the warning is heeded to and action taken, keeping in view the fact that the work involved in applying the method is largely based on judgement, knowledge of the process and technical skill in tracing down assignable causes of variation to their source.

This standard, was originally published in 1952, was largely a reproduction of the American Defence Emergency Standard Z 1.3-1942 'Control chart method of controlling quality during production' which had proved quite useful in the military stores purchases during second war. The first revision of this standard had been taken up with a view to make the standard more comprehensive by including control charts for medians and mid-ranges which are quicker, easy to operate and at the same time quite efficient for small samples. Further, the methodology of adopting control charts techniques for use when manufacturing is undertaken to a predetermined specification was also included. As the original standard was bulky, it was split into two parts during this revision, namely, Part 1 dealing with control charts for variables and Part 2 dealing with control charts for attributes.

This second revision of the standard has been taken up to include:

a) difference between assignable causes and chance causes in a tabular form,
b) choice of measuring equipment during the preliminaries to installation of control charts,
c) further guidance for deciding the frequency during the preliminaries to installation of control charts,
d) explanation for not considering lower control limit (*LCL*) for homogenization of range values in *R*-chart and further necessary actions,
e) conditions under which modified control chart should not be used,
f) many editorial corrections, and
g) amendment issued to this standard at appropriate place.

Further in this revision the text on process capability and its comparison with the tolerance have been suitably modified and reference has been given to IS 10645 : 2002 'Methods for estimation of process capability and process performance', is the necessary adjunct to this standard.

In addition to this Part 3, IS 397 has the four parts. The other parts are:

(Part 0) : 2003 Guidelines for selection of control charts (*first revision*)
(Part 2) : 2003 Control charts for attributes (*third revision*)
(Part 3) : 2003 Special control charts by variables (*first revision*)
(Part 4) : 2003 Special control charts by attributes (*first revision*)

The composition of the Committee responsible for the formulation of this standard is given in Annex D.

Indian Standard

METHODS FOR STATISTICAL QUALITY CONTROL DURING PRODUCTION

PART 1 CONTROL CHARTS FOR VARIABLES

(Second Revision)

1 SCOPE

1.1 This standard (Part 1) outlines the method of control chart by variables for controlling the quality during production. The principles of procedure pertaining to control charts for individual observations, averages, medians, mid-ranges, ranges and standard deviations are given in general terms.

1.2 The standard also lays down the procedures for the construction of modified control charts which can be usefully adopted when the manufacture is undertaken to predetermined specifications.

1.3 The principles of control charts have been illustrated with a variety of examples. Certain broad guidelines as to the interpretation of the data resulting from control charts are also included.

2 REFERENCES

The following standards contain provisions, which through reference in this text constitute provisions of this standard. At the time of publication, the editions indicated were valid. All standards are subject to revision and parties to agreements based on this standard are encouraged to investigate the possibility of applying the most recent editions of the standards indicated below:

IS No.	*Title*
7920	Statistical vocabulary and symbols:
(Part 1) : 1994	Probability and general statistical terms (*second revision*)
(Part 2) : 1994	Statistical quality control (*second revision*)
9300 (Part 2) : 1989	Statistical models for industrial applications: Part 2 Continuous models (*first revision*)
10645 : 1998	Method for estimation of process capability (*first revision*)
5420	Guide on precision of test methods:
(Part 1) : 1969	Principles and applications
(Part 2) : 1973	Inter laboratory testing

3 TERMINOLOGY

For the purpose of this standard the definitions given in IS 7920 (Part 1) and IS 7920 (Part 2) shall apply.

4 BASIC CONCEPTS UNDERLYING CONTROL CHART TECHNIQUE

4.1 Variation and Its Causes

In the repetitive making of a product so often met in the present day industrial production, variation in any chosen quality characteristic is an ever present phenomenon. Although such variation may be due to a variety of causes and their interaction known and unknown, which constantly influence the process, they can be broadly classified into following two distinct categories:

a) Variation due to assignable causes, such as, different settings of a machine, different batches of raw materials that are being fed, and changes in operators who have taken over in a new shift.

b) Variation due to chance (non-assignable or common) causes which are unavoidable in the process due to such inherent variation that exist in raw materials, machines, atmospheric conditions, etc.

4.2 Difference Between Chance Causes and Assignable Causes

Sl No.	*Chance (Non-assignable) Causes*	*Assignable Causes*
(1)	(2)	(3)
i)	Consists of many individual causes.	Consists of one or just a few individual causes.
ii)	Any one chance cause results in a minute amount of variation (but many chance causes act together to yield a substantial total).	Any one assignable cause can result in a large amount of variation.
iii)	Cannot economically be eliminated from a process.	Can be detected. Action to eliminate the cause(s) is usually economically justified.
iv)	When only chance cause is present, the process is operating at its best.	If assignable variation is present the process is not operating at its best.
v)	An observation within the control limits of random variation means the process should not be adjusted.	An observation beyond control limits usually means the process should be investigated and corrected.
vi)	Under chance causes, the process is sufficiently stable to use sampling procedures to predict the quality of total production or make process optimization studies.	With assignable causes present, the process is not sufficiently stable to use sampling procedures for prediction.

4.3 Process Under Statistical Control

When no assignable causes of variation are present in a process and it operates only under a system of non-assignable or chance causes, the process is said to be in a state of statistical control. Such variations due to chance causes occur in a random fashion and are usually found to obey certain statistical laws. If large number of observations obtained from a process, which is under a state of statistical control, are studied in the form of a frequency distribution, it will generally fall into a bell shaped symmetrical pattern wherein most of the observations cluster around the average value and fewer observations are found as one moves away from the average value.

4.4 Normal Distribution

4.4.1 The bell shaped symmetric pattern that is obtained by the observations emanating from a process under statistical control is generally well represented by the normal distribution. This distribution is characterized by two parameters, namely the mean value and the standard deviation. It is symmetrical and 99.73 percent of the observations lie within three standard deviations from the mean on both sides. Thus, less than 3 in a thousand observations are expected to fall beyond the three standard deviations from the mean on both sides. For further description about the properties of this distribution, attention in invited to IS 9300 (Part 2). Hence in a process under statistical control if any single measurement is taken and it does not fall within $\pm 3\sigma$ from the mean, it can be regarded as not belonging to that distribution or to that cause system and the presence of an assignable cause affecting the process may be inferred.

4.4.2 Just as a large number of individual measurements from a process follow a statistical law in the form of a known distribution, if samples of given size are taken from the same process at more or less regular intervals, then sample statistic, such as average or standard deviation, also follow known distributions. Of course, this distribution is not the same as the one obtained for the individual observations from the process. Generally speaking the distribution of the sample statistics that are dealt with in this standard are of the normal or known type whose parameters can be estimated.

4.5 Control Charts

4.5.1 The above mentioned behaviour of the sample statistics obtaining from a process influenced only by random causes is the foundation on which the control chart technique is based. Essentially it is a graphical method representing a sequence (in time) of sample statistics. It consists of a central line (*CL*) denoting the average value of the statistic being plotted and it has two control limits on either side of the central line which are called upper control limit (*UCL*) and lower control limit (*LCL*). The control limits are determined statistically from the probability distribution of the sample statistic.

4.5.2 The purpose of control chart is to obtain a state of statistical control by locating and eliminating the assignable causes and then to maintain the production in this state so as to ensure the manufacture of consistent products of acceptable quality. For this purpose the variation due to non-assignable causes is estimated and then used as the basis for the detection of the variation due to assignable causes by plotting the sample statistic on the control chart. As long as the plotted point is within the control limits, the process is left alone. However, if a point falls either below the lower control limit or above the upper control limit or depicts other unnatural pattern (*see* 7.2) there is a possibility of the existence of some assignable cause(s) and an investigation is made for taking action to eliminate such causes.

5 PRELIMINARIES TO THE INSTALLATION OF CONTROL CHARTS

5.1 Choice of Quality Characteristic

To start with, a decision has to be taken with regard to the quality characteristics for which a control programme is desired. Characteristics affecting the performance of the product should normally be the object of first attention. These may be the features of the materials used or components or parts of the products, for example, tensile strength of the core wire of cables or thickness of the insulation. Sometimes the characteristic may be for the finished product as a whole like the life of incandescent lamps.

5.2 Choice of the Place for Control

5.2.1 In any production process it is of utmost importance that proper checks through control charts are exercized at strategic points. To pinpoint the place for such controls it is desirable to determine the areas of maximum potential for return in the form of direct profits, reduction in scraps, increase in productivity, etc. A proper value analysis of all the performance may be quite helpful in the context.

5.2.2 It may also be worthwhile to study the production process to determine the nature and location of the causes that tend to give rise to deviations in the characteristic chosen. The method of inspecting individual article or product for the selected characteristic is equally important since factors like inspection fatigue may give rise to errors in observations. Thus, irregularities evident in quality data may arise from errors of inspection as well as from faults in the production process. Errors in inspection may result from a faulty test apparatus, faulty use of otherwise correct apparatus, etc. It is also to be decided beforehand whether the entire output of products should be considered as a single stream having a common system of causes or as two or more distinct streams each to be treated separately in the control programme because they come from different cause systems such as different conveyor lines, different machines or different shifts of workmen, etc.

5.3 Choice of Rational Sub-groups

5.3.1 Since the basic aim of control charts is to separate the variation due to assignable and non-assignable causes, it is evident that each sample should be representative of a homogeneous segment of the production flow. So in the ideal condition the variation found within the sample items should be due to non-assignable or random causes whereas the variation found between samples should be ascribable to some assignable causes. The division of the production flow in such a manner that each portion yields a sample having this property is known as 'rational sub-grouping'. A natural sub-group for example would be the output of a short time period since the variation in items manufactured close to each other in the time sequence are much more likely to represent chance fluctuations.

5.3.2 The problem of forming rational sub-groups also depends on the technical knowledge of the production process and the familiarity with the conditions under which the product is to be manufactured. Whereas it is not possible to give exact instructions for the formation of rational sub-groups that will cover all cases, a few illustrations may be helpful in this direction. Thus, if different machine settings have an effect on the quality characteristic that is being studied, all the units in a single sub-group should come from the same setting. Again, if different batches of material have an effect, then all units in one sub-group should be from the same batch. Extending this, it may generally be advisable not to form sub-groups such that a single sub-group will consist of items manufactured in different shifts, from components obtained from different sources, from different production lines, from different machines, moulds, operators, etc. In many situations a small sample taken in the order of production meets the principle of rational sub-groups, since it is likely to represent the immediate state of the process at the time sample was selected. However, it should be noted that this is not a universal recommendation. If one is taking sample from a machine with multiple spindles or multiple positions or heads, then a series of consecutive units from the machine as a whole will not form a rational sub-group capable of studying the variation between the different heads. For example, if a filling machine has six heads which simultaneously fill six consecutive containers in the production line, then every sixth unit taken (and not the consecutive six units) from the process will form a rational sub-group, because the variations within such sub-groups would be the inherent variation due to the heads and the variation between the sub-groups would be the variation obtaining from the different heads of the machine. In such situations, precise setting of the six heads becomes very crucial.

5.3.3 To avoid bias in the formation of rational sub-groups, the following two precautions must be taken in the selection of consecutive samples: (a) periodic selection should not coincide with any relevant periodic features in the process, and (b) selection of samples should not be made on a fixed time schedule if foreknowledge of the selection time would have an influence on the quality characteristic of the article selected.

5.4 Frequency and Size of Samples

5.4.1 No general rules may be laid down for frequency of sub-groups. Each case must be decided on its own merits considering both the cost of taking and analyzing measurements and the benefits to be derived from action based on control charts. In the initial use of the control chart for analyzing a process, it may be desirable to arrive at conclusions quickly by taking frequent samples. Later on, if troubles have been diagnosed and corrected and the function of the control chart has become the maintenance of the process control on current production, it may be advisable to reduce the frequency of sampling. As a guideline, sub-group frequencies for on going production monitoring could be twice per shift, hourly or some other feasible rate, however, interval between two sub-groups should be half of the known duration of trouble free performance.

5.4.2 The size of the sample to be taken depends on a number of practical considerations. However, generally speaking, large samples taken at less frequent intervals may detect a small shift in the process average more quickly, but small samples taken at more frequent intervals will detect a large shift more quickly. In most of the industrial use of the control charts, samples of size 4 or 5 are found to be quite common. A sample size of 5 is recommended since the computation of the average in this case can be considerably simplified. When median charts are used, samples of size 3 or 5 are most convenient since they avoid the computation altogether for the calculation of the median value. In case the mid-range is used, then also the computations are considerably simplified since the average of only two observations (largest and smallest) are to be obtained. It may, however, be noted that the use of median or mid-range in place of average results in a slight loss of efficiency but this loss is marginal in the case of small samples. Samples of smaller size may have to be used if the cost of measurements is too high. In most of the chemical industries using batch process the samples of size 1 or 2 are frequently preferred. It may, however, be noted that larger the sample size the more desirable it is to use the standard deviation rather than range as a measure of sub-group dispersions.

5.5 Choice of Measuring Equipment

The measuring equipment used for measuring the test results should be calibrated whose least count should be preferably 1/10th of tolerance or process capability. The variation due to measuring system should be quantified and minimized [*see* IS 5420 (Part 1) and IS 5420 (Part 2)]. This variation due to measuring system should be less than 10 percent of the total process variation of the characteristic. If this variation is between 10 percent to 30 percent, it may still be tolerable depending upon the application. If it exceeds 30 percent, the equipment should be regarded as inappropriate. In addition, the measurement uncertainty of the measuring equipment should be much less than the tolerance of the characteristic.

5.6 Choice of the Types of Control Charts

In the case of measurable quality characteristics it is generally the common practice to maintain a pair of control charts — one for the control of average level of the process (average chart or median chart or mid-range chart) and the other for the control of dispersion (range chart or standard deviation chart). Of course, when a chart for individual measurements is maintained, there is no possibility of keeping the companion chart for dispersion.

6 SETTING UP OF THE CONTROL CHART

6.1 Preliminary Data Collection

After having decided upon the quality characteristic which is to be controlled and the frequency and size of the sample to be taken, some initial inspection data has to be collected and analyzed for the purpose of determining the central line and control limits of the static. The preliminary data may be collected sample by sample (in sub-groups of size as decided in **5.4**) till about 25 samples are obtained from the continuous run of the production process. Care is to be exercised during the course of this initial data collection that the process is not unduly influenced intermittently by extraneous factors like change in the feed of raw materials, operators and machine settings.

6.2 Analysis of Preliminary Data

6.2.1 Analysis of the preliminary data is undertaken firstly by homogenizing the dispersion (range or standard deviation) of the various sub-groups and then by homogenizing the central tendency (such as average, median or mid-range). This order for homogenization process becomes necessary since the ranges (or standard deviation) also enter into the calculations needed for homogenizing the averages.

6.2.2 *Homogenization for Dispersion*

6.2.2.1 *Using range method*

If the sample size chosen is not more than 6, then for each of the sub-groups, the range (R) shall be first calculated and the average range value ($\bar{R}$) shall be computed from these ranges. Depending upon the sample size, the value of factor D_4 shall be chosen from Annex A. If all the ranges are found to be less than or equal to $D_4\ \bar{R}$ the initial data collected shall be deemed to be homogeneous and acceptable for the purpose of further calculation of the control limits. In case one or more range values are found to exceed the value $D_4 \bar{R}$, the observations in the sub-group corresponding to this range shall be discarded. For the remaining data, the above procedure shall be repeated (that is, the calculation of the new average range $\bar{R}$ and comparison of all the remaining ranges with $D_4 \bar{R}$) till all the range values are less than $D_4 \bar{R}$.

6.2.2.2 *Using standard deviation method*

In case the sample size chosen is fairly large, it is advisable to use standard deviation instead of the range since the later is less efficient for large samples. The procedure, however, is almost similar to that given in **6.2.2.1**. For each of the sub-groups, the standard deviation s shall be computed and from these values the average standard deviation $\bar{s}$ shall be computed. If all the individual s values are less than or equal to $B_4 \bar{s}$ (where B_4 is a factor chosen from Annex A for the corresponding sample size) then the initial data shall be considered to be homogeneous. However, if one or more s values are found to be exceeding $B_4 \bar{s}$ the observations corresponding to these sub-groups shall be discarded. For the remaining data the above procedure shall be repeated (that is, the calculation of a new average standard deviation $\bar{s}$ and the comparison of the remaining s values with $B_4 \bar{s}$) till all the standard deviation values are less than $B_4 \bar{s}$.

NOTE — *LCL* is not considered for the homogenization of ranges or standard deviations values as minimum variation in the process is intended and therefore is always welcome. The only point to be investigated at that stage is the correctness of this less value of range or standard deviation. If such a low value has really been obtained then such situations should be institutionalized for future.

6.2.2.3 In the above process of homogenization, if more than 25 percent of the sub-groups are discarded for being out of control, the entire set of data may be discarded. The fresh data should be collected after checking the process and eliminating the assignable cause(s) which were responsible for the high rate of outliers in the preliminary data.

6.2.3 *Homogenization for Central Tendency*

6.2.3.1 *Using average method*

From the homogenized data for dispersion (*see* **6.2.2.1** or **6.2.2.2**), the average of each sub-group ($\bar{X}$) shall be calculated. The average of these averages or the grand average ($\bar{\bar{X}}$) for all the sub-groups shall then be calculated. In case the data has earlier been homogenized by using ranges (*see* **6.2.2.1**) then a quantity $A_2 \bar{R}$ shall be calculated, where the factor A_2 is chosen from Annex A for the corresponding sample size. If any of the averages are found to lie outside the interval $\bar{\bar{X}} \pm A_2 \bar{R}$ then the observations in the sub-groups corresponding to these averages shall be discarded. From the remaining data, a fresh grand average shall be computed and the above procedure shall be repeated till all the average values are found to lie within $\bar{\bar{X}} \pm A_2 \bar{R}$. In case the data has earlier been homogenized by using standard deviation (*see* **6.2.1.2**), then instead of $A_2 \bar{R}$ the quantity $A_1 \bar{s}$ shall be used in the above homogenization process where the factor A_1 is suitably chosen from Annex A for the corresponding sample size.

6.2.3.2 *Using median method*

From the homogenized data for dispersion (*see* **6.2.1.1**), the median for each sample (M_e) shall be calculated and the average of these medians ($\bar{M}_e$) shall then be computed. A quantity $F_2 \bar{R}$ shall be calculated where the value of factor F_2 is chosen from Annex A depending upon the sample size. If any of the medians are found to lie outside the interval $\bar{M}_e \pm F_2 \bar{R}$ then the sub-groups corresponding to those medians shall be discarded. From the remaining sub-groups, a fresh average median value shall be computed and the above procedure shall be repeated till all the median values are found to lie within $\bar{M}_e \pm F_2 \bar{R}$.

6.2.3.3 *Using mid-range method*

The process of homogenization in this case is similar to that for the homogenization of the medians except that the mid-range (*M*) has to be calculated for each sub-groups (instead of median) and the factor F_2 has to be replaced by G_2. The value of factor G_2 is chosen from Annex A depending upon the sample size.

6.3 Control Limits

6.3.1 *Control Limits for Measures of Dispersion*

6.3.1.1 *Range (R) chart*

The central line for the range chart is given by the homogenized average range value ($\bar{R}$) obtained according to **6.2.2.1**. The upper control limit (*UCL*) and the lower control limit (*LCL*) for the range chart are obtained as $D_4 \bar{R}$ and $D_3 \bar{R}$ where values of factors D_4 and D_3 are obtained from Annex A depending upon the sample size.

NOTE — *LCL* for the range chart coincides with the *X*-axis as $D_3 = 0$.

6.3.1.2 *Standard deviation (s) chart*

The central line for the standard deviation chart is given by the homogenized average standard deviation value

($\bar{s}$) obtained according to **6.2.2.2**. The upper control limit (*UCL*) and the lower control limit (*LCL*) for the standard deviation chart are obtained as $B_4\,\bar{s}$ and $B_3\,\bar{s}$ where values of the factors B_4 and B_3 are chosen from Annex A depending upon the sample size.

6.3.2 *Control Limits for Central Tendency*

6.3.2.1 *Average ($\bar{X}$) chart*

The central line for the average chart is the grand average $\bar{\bar{X}}$ calculated from the homogenized averages as indicated in **6.2.3.1**. In case the range has been used in the earlier homogenization process, then *UCL* and the *LCL* for the average chart are obtained as follows:

$$UCL = \bar{\bar{X}} + A_2\,\bar{R} \quad \text{and} \quad LCL = \bar{\bar{X}} - A_2\,\bar{R}$$

The value of the factor A_2 is suitably chosen from Annex A depending upon the sample size.

In case, standard deviation has been used in the earlier homogenization process, then *UCL* and *LCL* for the average chart are obtained as follows:

$$UCL = \bar{\bar{X}} + A_1\bar{s} \quad \text{and} \quad LCL = \bar{\bar{X}} - A_1\bar{s}$$

The value of the factor A_1 is chosen from Annex A depending upon the sample size.

NOTE — When the upper specification limit (U) and the lower specification limit (L) for the characteristic are stipulated, it may be better to have the central line for the average chart at $(U + L)/2$ for control purposes instead of using the homogenized grand average value. However, for further details a reference is invited to **6.4**.

6.3.2.2 *Median (M_e) chart*

The central line for the median chart is the average of the homogenized median values ($\bar{M}_e$) and the control limits are obtained as follows:

$$UCL = \bar{M}_e + F_2\,\bar{R} \quad \text{and} \quad LCL = \bar{M}_e - F_2\,\bar{R}$$

The value of the factor F_2 is chosen from Annex A depending upon the sample size.

6.3.2.3 *Mid-range (M) chart*

The average of the homogenized mid-range ($\bar{M}$) values is the central line for the mid-range chart. The *UCL* and the *LCL* for the mid-range chart are obtained as follows:

$$UCL = \bar{M} + G_2\,\bar{R} \quad \text{and} \quad LCL = \bar{M} - G_2\,\bar{R}$$

The value of factor G_2 is chosen from Annex A depending upon the sample size.

6.3.3 *Control Limits for Individual Control Chart*

6.3.3.1 When variation within the sub-group is expected to be negligible, for example, in liquid formulations or powder or when the cost of inspection is prohibitive; it may not be desirable to have more than one sample from each sub-group. In such situations, individual control chart is used. As in such cases, the sample size would be one, the range value for each sub-group can not be calculated and therefore the homogenization procedure recommended in **6.2.2.1** is not applicable. Hence, after collecting the initial observations (numbering 25 or more) the method of moving range is used. For this purpose the successive differences of the individual values, that is, the difference between the first and the second observation, between the second and the third observation, and so on shall be calculated, ignoring the sign of the difference. Thus, for 25 observations, there will be 24 values of the moving range. From these moving ranges the value of the average moving range $\bar{R}$ is calculated. If each of the individual moving range (R) is less than or equal to 3.267 $\bar{R}$ then the preliminary data is considered to be homogeneous. If, however, one or more range values exceed this limit, then those ranges shall be eliminated and the procedure shall be repeated till all the ranges are found to be homogeneous.

6.3.3.2 From the entire data, overall average ($\bar{X}$) shall be computed. The *UCL* and the *LCL* for individual chart are obtained as $\bar{X} + 2.66\ \bar{R}$ and $\bar{X} - 2.66\ \bar{R}$ where $\bar{R}$ is the average of the homogenized moving ranges. If any of the individual values are found to lie outside the control limits, they shall be eliminated and the new average shall be calculated from the remaining observations. This process shall be repeated till all the remaining observations are found to be within the control limits.

6.4 Control Charts Based on Known Standard Values

6.4.1 In some cases there may not be any need for the collection and analysis of preliminary data as indicated in **6.1** and **6.2**, if from the past knowledge or records the standard values of the parameters of the process are known—the process average to be maintained at a level μ and the standard deviation of the process realistically estimated as σ. While controlling these characteristics, for which both an upper specification limit (U) and a lower specification limit (L) are stipulated, it is advantageous to control the process at an average level $\mu = (U + L)/2$. Even in the cases where initial control chart had been set up on the basis of collection and analysis of preliminary data, after certain lapse of time when the process had stabilized and is in a state of statistical control, it becomes necessary to re-evaluate the parameters used in the calculation of control limits.

6.4.2 The value of process average (μ) and process standard deviation (σ) normally based on about hundred or more points plotted on the initial control charts may be taken as the standard values for the re-calculation of the control limits. Using the standard values, the control limits are calculated as given in **6.4.3**.

NOTE — In case the initial control limits are based on average range ($\bar{R}$), then a good estimate of the process standard deviation is obtained as $\sigma = \bar{R}/d_2$ where $\bar{R}$ is based on hundred or more sample ranges and d_2 is a suitable factor taken from Annex A depending upon the sample size.

6.4.3 *Control Limits for Measures of Dispersion*

6.4.3.1 *Control limits for range chart*

The central line for the range chart is placed at $d_2\sigma$ and the *UCL* and *LCL* are drawn at $D_2\sigma$ and $D_1\sigma$ respectively where the factors d_2, D_2 and D_1 are suitably chosen from Annex A depending upon the size of the sample.

6.4.3.2 *Control limits for standard deviation chart*

The central line for the standard deviation chart is placed at $c_2\sigma$ and the *UCL* and *LCL* are placed at $B_2\sigma$ and $B_1\sigma$ respectively where values for the factors c_2, B_2 and B_1 are chosen from Annex A depending upon the sample size.

6.4.4 *Control Limits for Measures of Central Tendency*

6.4.4.1 *Control limits for average chart*

The central line for the average chart is drawn at μ, and *UCL* and *LCL* are drawn at $\mu + A\sigma$ and $\mu - A\sigma$ respectively where the value for factor *A* is chosen from Annex A depending upon the sample size.

6.4.4.2 *Control limits for median chart*

The central line of the median chart is drawn at μ and *UCL* and *LCL* are drawn at $\mu + F\sigma$ and $\mu - F\sigma$ respectively where the value for factor *F* is chosen from Annex A depending upon the sample size.

6.4.4.3 *Control limits for mid-range chart*

The central lie for the mid-range chart is drawn at μ, and *UCL* and *LCL* are drawn at $\mu + G\sigma$ and $\mu - G\sigma$ respectively, where the value for factor *G* is chosen from Annex A depending upon the sample size.

6.4.4.4 *Control limits for individual chart*

The central line for the individual chart is placed at μ, and the *UCL* and *LCL* are drawn at $\mu + 3\sigma$ and $\mu - 3\sigma$ respectively.

6.5 Some Practical Guidelines for Drawing the Control Charts

6.5.1 On a suitable form or cross-sectional or graph paper the control charts are normally drawn. The horizontal scale indicates the sub-group number (possibly designated by the date and sample number) whereas statistical measures chosen such as average, median, range, etc, are represented on the vertical scale. It is usually satisfactory to use millimetre graph paper wherein the sub-group numbers are indicated on the horizontal scale at intervals of not less than 0.5 cm. Whenever charts for central tendency (average, median or mid-range) and dispersion (standard deviation or range) are simultaneously maintained, the chart for central tendency is placed at the top and the one for measure of dispersion is placed below using the same horizontal and vertical scales. Every care should be taken to exactly align the horizontal axis of the two graphs. Thus for each sub-group two points should be simultaneously available which are plotted on the respective control charts on the same vertical line.

6.5.2 A typical proforma for collection of data is given at Annex B.

7 MAINTENANCE OF CONTROL CHARTS

7.1 Using the Control Chart During Production

7.1.1 The control charts have to be displayed in a conspicuous place where they may be readily viewed by those concerned, such as, operator, foreman, superintendent, chief inspector and quality control engineer.

7.1.2 There should be no delay in plotting all the points on the control chart after the sample has been taken.

7.1.3 When a point falls outside the control limits, the type of the action to be taken should be predetermined. Two kinds of actions may be involved, namely, action on the lot of product and the action on the process. In the case of action on the lot of products, examination of additional samples from the lot with a view to determining whether it should be held up or released may be pertinent. The action on the process may consist of investigating the suitability of raw materials, the correctness of the testing procedure, etc, so that the cause for the lack of control is identified and if possible, corrective action is taken immediately to prevent its recurrence.

7.1.4 After the causes of trouble are located and eliminated, the control limits have to be periodically reviewed and altered. A definite schedule for such reviews is to be set up and followed as a routine manner, for example, every 50 or 100 points, every month, etc, depending on the frequency of sampling and the degree of control shown by the charted history. While reviewing the control limits, it may be noted that, as far as possible, they should not become wider than the earlier limits obtained as this will affect the better process control achieved earlier.

7.1.5 Brief notes may be added on the control chart regarding the causes of trouble found in the manufacturing process or inspection methods and the corrective action taken. In fact it is desirable to record the changes taking place in the inputs which may be useful in identifying the assignable causes for corrective action. The completed control charts may be kept in the permanent file for a specified useful period as a record of the quality of the product.

7.2 Detection of Assignable Causes

7.2.1 Since the basic aim of the control chart is to detect the presence of any unnatural pattern in the process of manufacture, which will have to be removed, it is useful to have an idea about the following characteristics of a natural pattern:

a) Majority of the points are near the central line,
b) A few of the points are spread out and approach the control limits, and
c) None of the points (or at least only a very rare and occasional point) exceeds the control limit.

7.2.2 There are different types of unnatural patterns which may be noticed in the control chart. The ease and the frequency with which an operator will be able to spot any unnatural pattern will depend on his experience in running the control chart as also his knowledge of the process. However, the following are some of the unnatural patterns, which occur more often:

a) *Instability* — The presence of points outside the control limits.
b) *Stratification* — Up and down variations are very small in comparison with the width of the control limits and absence of points near the control limits.
c) *Mixture* — A tendency to avoid the central line with too many points near the control limits.
d) *Cyclic or systematic system* — A long series of points which are high, low, high, low without any interruption in this regular sequence.
e) *Trend* — A series of consecutive points (seven) without a change in direction.

7.2.3 The above unnatural pattern may be observed when there is lack of control in:

a) Average (or median or mid-range) chart only,
b) Range (or standard deviation) chart only, and
c) Both the average and range charts.

7.2.3.1 *Lack of control in the average chart only*

This is the common type of lack of control observed in manufacturing wherein a shift in the process average occurs with little or no changes in the process dispersion. In such cases the control chart is often of great value to the machine setter to help him to centre the machine setting in order to produce a desired process average. This type of lack of control is shown on the average chart. Unless the changes in the process average take place within a sub-group, the range chart will show control. Since the control limits are set far enough from the central line on the chart with the possibility of very few points outside the limit without a real change in the process, small shifts in the process average will not cause many points to fall out of control. Sufficient grounds exist for suspicion that the process average has shifted when:

a) 7 successive points on the control chart are on the same side of the central line,
b) 10 out of 11 successive points are on the same side of the central line,
c) 12 out of 14 successive points are on the same side of the central line,
d) 14 out of 17 successive points are on the same side of the central line, and
e) 16 out of 20 successive points are on the same side of the central line.

7.2.3.2 *Lack of control in the dispersion chart only*

The inherent variability of a process may change from time-to-time even though there is no change in the process average. For any process where the skill and care of the operator is an important factor, the common cause of increase in variability is a change from one operator to another who is less skilful or less careful. In fact an operator's skill and care may also vary from day-to-day or from hour-to-hour. Extreme runs above the central line on the range chart also give strong evidence of lack of control in the process variability. Generally speaking, variability of a process dispersion is particularly likely to be found in those processes where the skill of the operator is important. Hence the first step in improving such processes should be an attempt to bring the process dispersion into statistical control. The value of the average range ($\bar{R}$) used for the calculation of the control limits should be reviewed from time-to-time since it has a direct bearing on the calculation of the control limits for both the central tendency and dispersion.

7.2.3.3 *Changes in both process average and dispersion*

When the process dispersion as well as the process average are shifting, it is obvious that lack of control will be indicated in both the range chart and the average chart. This state of affairs is generally found in the initial stages of the use of control chart for variables for analysis of many manufacturing operations. Where several assignable causes of variation exist, the elimination of some of the causes will decrease the number of out-of-control points but will not eliminate all of them. In such circumstances, one should not be discouraged by the continuance of some points out of control. On the other hand the chart should be viewed as an indication that further improvement is possible and as an incentive to keep hunting for more sources of trouble.

It is also worthwhile examining whether an error in measurement may be an assignable cause of variation in the values resulting from the measurements, because an error in setting a measuring device may make apparent sudden shifts in process average. Frequent errors in setting may also make irregular shifts in the average. Some type of wear of measuring device may cause increase in the process dispersion. Yet other types of wear may give rise to trends in averages.

8 CONTROL LIMITS AND SPECIFICATION LIMITS

Since most of the control charts, like average chart and range chart are to be maintained in the production operations for the detection of assignable causes of variation, a common source of confusion is the existence of specification limits side-by-side. It should not be confused that the control limits for average chart are the same as the specification limits. Control charts enable one to control process compatible with its capability. In fact, the specification limits generally apply to the individual values whereas the control limits of the average chart apply to the distribution of the average. For this reason it is always advisable that specification limits are not drawn on the average chart. However, it should not be presumed that the control limits and specification limits are not related in any manner.

NOTE — Only in the case of control charts for individuals (*see* **6.3.3** or **6.4.4.4**), the specification limits may be indicated on the chart itself for comparison purposes.

9 ESTIMATION OF PROCESS CAPABILITY

From a process which is under statistical control for a reasonably long time, a good estimation of the process capability may obtained from the control chart for dispersion of the process (standard deviation chart or range chart). In case the standard deviation chart is maintained, the process capability is obtained as $6\,\bar{s}/c_2$ where value of c_2 is given in Annex A depending upon the size of the sample. In the case of range chart, the estimate is obtained as $6\,\bar{R}/d_2$ where value of d_2 is chosen from Annex A depending upon the sample size. If, however, the process standard deviation (σ) is known beforehand, the process capability is estimated as $6\,\sigma$. In physical terms, the estimate of the process capability gives an upper limit to the variation that can be found in all the items emanating from the process as long as it is kept in the state of statistical control. For further details of estimation of process capability, reference may be made to IS 10645.

10 SETTING SPECIFICATION LIMITS AND INSTALLATION OF MODIFIED CONTROL LIMITS

10.1 In a realistic situation where the specification limits are drawn after a thorough study of the manufacturing process, the difference between the upper specification limit (*U*) and the lower specification limit (*L*) should not vary unduly from the process capability as estimated in **9**. But in many of the existing situations, specification limits are set in advance for the manufacture of an item on the basis of engineering, technological or other practical considerations. In such cases, after the manufacture has commenced and the process is studied and found to be in a state of statistical control, one of the following three situations may arise.

The tolerance difference between the upper and lower specification limits is:

a) Smaller than the estimate of process capability,
b) Of the same order as the estimate of process capability, and
c) Larger than the estimate of process capability.

The probable actions to be taken in each of the above three situations have been discussed in IS 10645.

10.2 When the difference between the upper and the lower specification limits (*U–L*) is larger than the estimated process capability, then one of the probable actions recommended in IS 10645 is the use of modified control chart.

10.3 The adoption of the modified control limits has many economic advantages since it permits limited shifts in the process average, thereby avoiding the unnecessary stoppage of production to hunt for trouble whenever the shifts in process average are not sufficient to cause the production of defective products. Therefore, this chart may be particularly useful when applied to intermittent short production runs in machining operations where the process capability has been determined from previous runs. The more (*U–L*) exceeds the estimated process capability the greater the permissible latitude in machine setting. The use of modified control limits may simplify the problem of maintaining machine settings that are good enough for practical purposes. The modified control limits have also been found useful when a process is subject to tool wear, since the natural spread of the process at any time is much less than the spread over the whole life of the tool.

10.4 Modified control chart, however, is not recommended when components are meant for assembly or final finishing operations or clearance is important. Further, modified control limits are workable as long as the estimate of the process capability remains constant. In other words the process dispersion must remain in statistical control. Wherever process dispersion behaves erratically, modified control limits are not appropriate. For this reason a chart for range (or standard deviation) should accompany any average chart using modified control limits.

10.5 The upper and lower control limits for modified control charts are placed within the range of upper and lower specification limits in such a manner that the control limits are equidistant from the specification limits in the following manner:

$$UCL = U - V_1\sigma$$

$$LCL = L + V_1\sigma$$

where the value of factor V_1 is suitably chosen from Annex C depending upon the sample size. In case the control charts are maintained after calculating the average range ($\bar{R}$), the upper and lower control limits are obtained as follows:

$$UCL = U - V_2\,\bar{R}$$

$$LCL = L + V_2\,\bar{R}$$

The value of V_2 is chosen from Annex C depending upon the sample size. The central line in both the above cases is drawn at $(U + L)/2$.

11 ILLUSTRATIVE EXAMPLES

11.1 Example for Installing Control Charts for Average and Range

11.1.1 In a firm manufacturing cuprous oxide dusting powder by continuous process, the average daily production was around 25 tonnes (for a shift of 8 h). The product was packed in bags of 50 kg each so that in a day about 500 bags were filled for despatch. Since the cuprous oxide content (percent by weight) is an important characteristic, which has to be controlled, it was felt desirable to install suitable control charts (average and range) for the purpose.

11.1.2 Five consecutive bags filled at the final stage were considered to constitute a desirable sample size and on practical considerations it was agreed that a frequency of sampling of once in every two hours would be adequate. After taking proper care with regard to the homogeneity of raw materials, mixing, machine maintenance, etc, the initial data as given in Table 1 and also plotted in Fig. 1 spread over a week were collected.

11.1.3 As a first step for the installation of control charts, the ranges were homogenized as follows:

11.1.3.1 From the 25 ranges (*see* col 11 of Table 1) the average range was calculated as:

$\bar{R} = 8.58/25 = 0.34$

11.1.3.2 Upper control limit (UCL) for the range chart is then calculated as:

> $D_4\bar{R} = 2.115 \times 0.34 = 0.72$, the value of D_4 being taken from Annex A for a sample of size 5. It was found that the range for sub-group number 11 was outside UCL and so this sub-group was discarded.

11.1.3.3 The average range calculated from the remaining 24 sub-groups was obtained as:

> $\bar{R} = 7.84/24 = 0.33$ and $D_4\,\bar{R} = 2.115 \times 0.33 = 0.70$. Again the range for the sub-group number 9 was outside the UCL and hence this sub-group was also discarded.

11.1.3.4 The average range for the remaining 23 sub-groups was obtained as:

> $\bar{R} = 0.31$ and $D_4\,\bar{R} = 2.115 \times 0.31 = 0.65$. It was found that all the 23 ranges were now less than UCL and hence the average range value ($\bar{R}$) = 0.31 was taken as homogenized range value for the purpose of drawing the control limits. The lower control limit for the range chart was now obtained as $D_3\,\bar{R} = 0$ as the value of D_3 for sample size 5 is zero.

11.1.4 For the 23 samples, which were found to be homogeneous with regard to range, the grand average $\bar{\bar{X}}$ was calculated as 10.05 and $A_2\,\bar{R} = 0.58 \times 0.31 = 0.18$, where the value of factor $A_2 = 0.58$ is chosen from Annex A for a sample of size 5. The control limits for the average chart were then obtained as $\bar{\bar{X}} \pm A_2\,\bar{R}$ which came out as 10.23 and 9.87. It was found that for the sub-groups number 3, 5, 6 and 7 the average values fell outside the control limits. This perhaps could be explained by the fact that there was some erratic behaviour in the amount of cuprous oxide content when the programme for the installation of control charts was initiated by collecting suitable samples. Hence these 4 sub-groups were discarded and a new grand average was calculated as $\bar{X}$ = 10.04 leading to the control limits of 10.22 and 9.86. It was then found that all the remaining 19 sub-groups were within the control limits and these limits were taken for the installation of average chart.

NOTE — The elimination of the four points falling outside the average chart has resulted in the proper centering of the process by bringing down the grand average from 10.05 to 10.04.

11.1.5 After the installation of the average and range charts, further data was regularly plotted in these charts at 2 h intervals on subsequent dates. A typical extract of such a data is given in Table 2 and also plotted in Fig. 2. It may be noticed that the point of the average chart corresponding to sub-group number 3 has fallen below the lower control limit indicating that there may be an assignable cause worth investigating. Accordingly the foreman in charge of the production who did a thorough investigation, found that the low average value of the technical content was due to zero error in the balance used for weighing the technical content before mixing. This error was suitably rectified and thereafter the process was found to be in statistical control.

11.2 Example for Installing Modified Control Charts

11.2.1 In the earlier example given in **11.1** if the firm were to produce cuprous oxide dusting powder to the nominal value of 10 percent (by weight) with a tolerance of – 5 to + 10 percent then the upper specification limit for cuprous oxide content would be 11.00 percent whereas the lower specification limit would be 9.50 percent. If the process is controlled at the levels obtained in **11.1** then the over all average range for the entire period works out as $\bar{R} = 15.89/43 = 0.37$. Hence an estimate of the process capability is obtained as $6\ \bar{R}/d_2 = 0.95$. It may be noticed that this estimate of process capability is much smaller than the difference between the upper and lower specification limits U–L = 1.50 percent. Hence the firm decided to install the modified control chart for averages. The upper and lower control limits for the modified chart were then calculated as follows:

$$UCL = U - V_2\ \bar{R} = 11.0 - 0.713 \times 0.37$$
$$= 11.00 - 0.26 = 10.74$$
$$LCL = L + V_2\ \bar{R} = 9.50 + 0.713 \times 0.37$$
$$= 9.50 + 0.26 = 9.76$$

11.2.2 Thus, the new control limits for average were from 9.76 to 10.74 percent as compared to the earlier limits of 9.86 percent to 10.22 percent. This wider range for the control chart for averages greatly helped the firm in meeting the stipulated specifications without unduly worrying about the permissible shifts in the process average value thereby leading to economy in production.

11.3 Example for Installing Individual Control Charts

11.3.1 In a firm, the oil of turpentine of Grade I was being manufactured on a batch process. Specific gravity was one of the important characteristics of the oil which had to be controlled properly. However, due to limitations of testing, only one composite sample (collected from different portions of the materials in a batch) could be analyzed for a batch and so it was decided to install a control chart for individuals.

11.3.2 The collection of the initial data spread over the production of 22 batches is given in Table 3. From the 22 results obtained for 22 batches, moving ranges of two successive values were computed. Thus there were 21 moving range values obtained from 22 results. The average range $\bar{R}$ was then calculated as $\bar{R} = 0.064/21 = 0.003$. For the homogenization of the ranges the upper control limit for the range chart was then calculated as $D_4\ \bar{R} = 3.267 \times 0.003 = 0.009\ 8$. Since all the range values were found to be less than this upper limit, this average range value of 0.003 was used for the construction of control chart for individuals. The *UCL* and the *LCL* for the individual chart were obtained as $\bar{X} \pm 2.66\ \bar{R} = 0.859 \pm 0.008$ leading to the limits 0.851 to 0.867. The individual values got by analyzing the composite sample for each batch were also plotted in Fig. 3. It would be seen that all the individual values were within the control limits. This indicated that the process was in a state of statistical control.

11.3.3 The specification limits for specific gravity of oil of turpentine (Grade I) were given as 0.852 to 0.862. Unlike in the case of averages where the indication of the specification limits on the control charts is strictly forbidden, in the case of individual chart it would be always advantageous to draw the specification limits along with control limits so as to find out how the process is behaving with regard to the specification limits. Specification limits were hence drawn in the control chart for individuals. It would be noticed that the lower specification limit was just above the lower control limit whereas upper specification limit was lower than the upper control limit. As a result three values out of 22 were found to be lying outside the upper specification limit although they were within the upper control limit. An estimate of the process capability is obtained as $6\ \bar{R}/d_2 = 0.016$. This is in excess of the difference between the upper and lower specification limits $(U - L)$ which comes out to 0.010 This indicated the need for either lowering the variation within the process or modifying the specification limits.

Table 1 Control Chart Data Sheet (Initial Data)

(*Clauses* 11.1.2 *and* 11.1.3.1)

Product:	Cuprous oxide dusting powder	Production Order No.:	
Characteristics:	Cuprous oxide content (percent by weight)	Normal Daily Output:	25 tonnes
Unit of Measurement:	Weight by percentage	Sample Size:	5
Nominal Value:	10 percent	Frequency:	Once in every 2 h
Specified Tolerance:	+10 percent of the nominal value – 5 percent of the nominal value	Inspector:	
Specification No.:	IS 1669		

Sub-group No.	Date	Time	Percentage of Cuprous Oxide Content in the Sample Bag					Total	Average	Range	Remarks
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1	July 1	830	9.99	9.6	9.8	10.05	9.91	49.35	9.87	0.45	
2	July 1	1030	10	10.29	10.14	10.07	10.14	50.64	10.13	0.29	
3	July 1	1230	10.34	10.43	10.24	10.43	10.34	51.78	10.36	0.19	
4	July 1	1430	9.8	10.14	10.24	9.99	10.24	50.41	10.08	0.44	
5	July 2	830	10.27	10.21	10.44	10.39	10.39	51.7	10.34	0.23	
6	July 2	1030	9.63	9.75	9.99	9.87	9.75	48.99	9.8	0.36	
7	July 2	1230	9.86	9.77	9.77	9.96	9.96	49.32	9.86	0.19	
8	July 2	1430	9.97	10.16	10.16	10.07	10.26	50.62	10.12	0.29	
9	July 3	830	9.88	10.39	9.68	10.14	10.05	50.14	10.03	0.71	
10	July 3	1030	9.88	9.78	9.78	9.98	9.98	49.4	9.88	0.20	
11	July 3	1230	9.68	10.42	10.23	9.86	9.97	49.96	9.99	0.74	
12	July 3	1430	10.35	10.12	10.12	9.99	10.35	51.04	10.21	0.36	
13	July 4	830	10.23	9.99	10.23	10.35	10.35	51.04	10.21	0.36	
14	July 4	1030	9.87	10.12	10.08	9.99	10.12	50.33	10.07	0.36	
15	July 4	1230	9.9	9.96	9.78	10.14	10.2	50.1	10.02	0.24	
16	July 4	1430	9.72	9.84	10.02	10.02	10.08	49.44	9.89	0.36	
17	July 5	830	9.96	10.07	10.1	10.17	10.12	50.34	10.07	0.21	
18	July 5	1030	9.81	9.95	9.85	9.9	10.2	49.96	9.99	0.39	
19	July 5	1230	10.05	9.90	10	10.05	10.15	50	10	0.30	
20	July 5	1430	10.29	10.10	9.81	10.05	10.05	50.49	10.1	0.29	

Table 1 — *Concluded*

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
21	July 6	830	9.9	10.05	10.25	9.95	10	49.71	9.94	0.24	
22	July 6	1030	10.1	9.96	9.91	9.86	10.06	50.23	10.05	0.39	
23	July 6	1230	9.81	10.16	10.16	10.1	10.06	50.04	10.01	0.35	
24	July 6	1430	9.96	10.01	10.21	9.91	10.02	50.11	10.02	0.35	
25	July 7	830	10.3	10.21	9.96	10.16	10.02	50.65	10.13	0.34	
							Total		251.17	8.58	

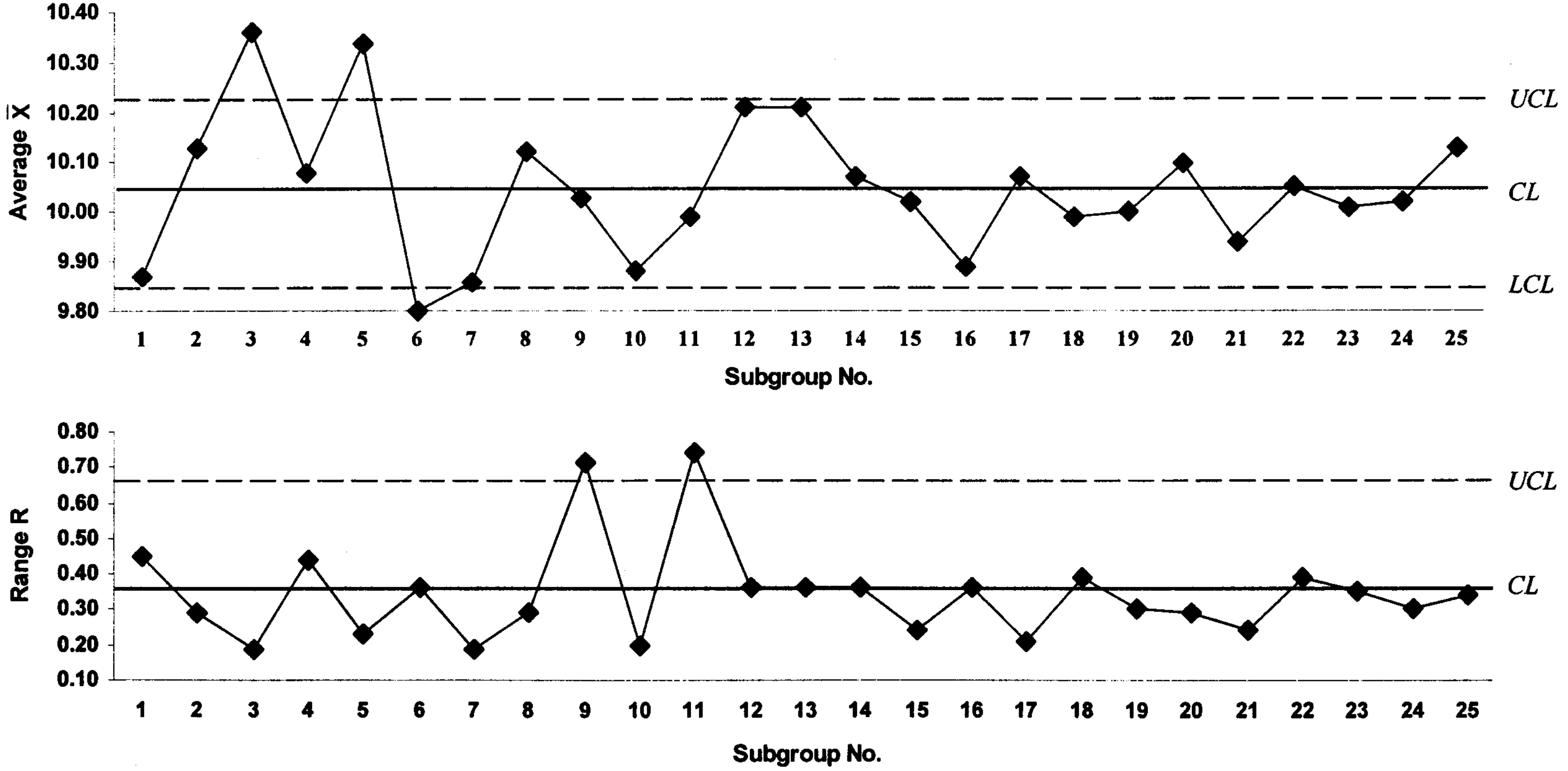

FIG. 1 AVERAGE AND RANGE CHARTS (INITIAL DATA)

Table 2 Control Chart Data Sheet (Subsequent Data)

(*Clause* 11.1.5)

Product:	Cuprous oxide dusting powder	Production Order No.:	
Characteristics:	Cuprous oxide content (percent by weight)	Production Department No.:	
Unit of Measurement:	Weight by percentage	Normal Daily Output:	25 tonnes
Nominal Value:	10 percent	Sample Size:	5
Specified Tolerance:	+10 percent of the nominal value – 5 percent of the nominal value	Frequency:	Once in every 2 h
		Inspector:	
Specification No.:	IS 1969		

Sl No.	Sub-group No.	Date	Time	Percentage of Cuprous Oxide Content in the Sample Bag					Total	Average	Range	Remarks
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
i)	1	1 August	830	10.09	10.39	10.09	9.78	10.19	50.54	10.11	0.61	
ii)	2	1 August	1030	9.68	9.88	9.98	10.09	9.78	49.41	9.88	0.41	
iii)	3	1 August	1230	9.82	9.63	9.80	9.76	10.11	49.12	9.82	0.48	
iv)	4	1 August	1430	9.88	9.88	10.39	10.39	9.18	50.42	10.08	0.51	
v)	5	2 August	830	10.29	10.09	9.88	10.09	10.09	50.64	10.13	0.41	
vi)	6	2 August	1030	9.78	10.29	10.19	10.29	9.98	50.53	10.11	0.51	
vii)	7	2 August	1230	10.09	10.09	10.29	9.88	9.78	50.13	10.03	0.51	
viii)	8	2 August	1430	10.09	9.88	10.19	9.88	10.09	50.13	10.03	0.31	
ix)	9	3 August	830	10.04	9.87	9.99	10.09	9.81	49.80	9.96	0.28	
x)	10	3 August	1030	9.77	10.05	10.18	9.85	9.92	49.77	9.95	0.41	
xi)	11	3 August	1230	10.10	9.87	10.03	10.11	9.89	50.00	10.00	0.24	
xii)	12	3 August	1430	10.03	9.84	10.31	10.01	10.21	50.40	10.08	0.47	
xiii)	13	4 August	830	10.08	10.11	10.24	10.12	10.10	50.65	10.13	0.16	
xiv)	14	4 August	1030	9.86	9.90	10.02	9.85	9.92	49.55	9.91	0.17	
xv)	15	4 August	1230	10.18	10.29	10.11	9.92	9.90	50.40	10.08	0.39	
xvi)	16	4 August	1430	10.14	10.20	10.08	10.17	9.94	50.53	10.11	0.26	

Table 2.— *Concluded*

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
xvii)	17	5 August	830	10.19	9.84	9.81	10.15	10.19	50.18	10.04	0.38	
xviii)	18	5 August	1030	9.69	9.84	9.70	10.02	10.19	49.44	9.89	0.50	
xix)	19	5 August	1230	10.06	9.72	10.27	9.97	9.90	49.92	9.98	0.55	

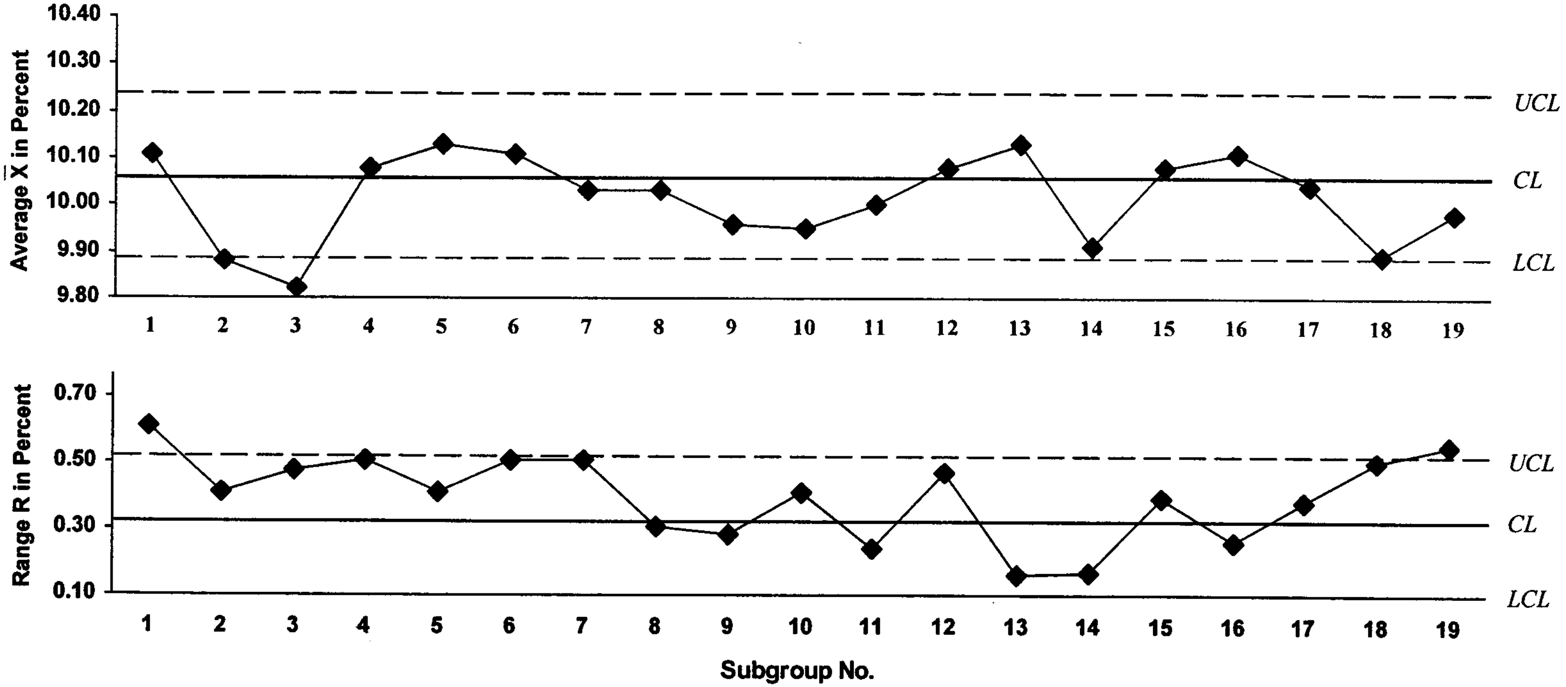

FIG. 2 AVERAGE AND RANGE CHARTS (SUBSEQUENT DATA)

11.4 Example for Installing Control Charts for Median and Range

11.4.1 *Control Chart for Median and Range* (*see* Table 4 and Fig. 4).

11.4.2 *Range Chart*

$$\bar{R} = 22.0/25 = 0.888$$

$$UCL = D_4 \bar{R} = 2.115 \times 0.888 = 1.88$$

As range hour No. 20 is greater than *UCL*, this range value is deleted for homogenization and calculations are done again.

$$\bar{R} = \frac{22.2 - 2.0}{24} = \frac{20.2}{24} = 0.84$$

$$UCL = 2.115 \times 0.84 = 1.78$$

Since all the values of range are within *UCL*, this value of $\bar{R} = 0.84$ is taken as homogenized range.

11.4.3 *Median Chart*

$$\bar{M}_e = \frac{1\,264.6 - 52.3}{24} = \frac{1\,212.3}{24} = 50.51$$

$$UCL = \bar{M}_e + F_2 \bar{R} = 50.51 + 0.691 \times 0.84$$
$$= 50.51 + 0.58 = 51.09$$

$$LCL = \bar{M}_e - F_2 \bar{R} = 50.51 - 0.58$$
$$= 49.93$$

Since the median values for hour number 18 and 19 are more than *UCL*, these values are deleted for homogenization.

$$\bar{M}_e = \frac{1\,212.3 - 52.1 - 52.3}{22} = \frac{1\,107.9}{22} = 50.36$$

$$UCL = 50.36 + 0.58 = 50.94$$

$$LCL = 50.36 - 0.58 = 49.78$$

Since all the median values are within *LCL* and *UCL*, these are taken as the control limits for median chart.

Table 3 Values of Specific Gravity of Composite Samples Analyzed for Different Batches

(*Clause* 11.3.2)

Sl No. (1)	Batch No. (2)	Specific Gravity Value (3)	Moving Range of the Two (4)
i)	1	0.861	–
ii)	2	0.860	0.001
iii)	3	0.861	0.001
iv)	4	0.865	0.004
v)	5	0.861	0.004
vi)	6	0.860	0.001
vii)	7	0.862	0.004
viii)	8	0.858	0.004
ix)	9	0.860	0.004
x)	10	0.859	0.001
xi)	11	0.864	0.005
xii)	12	0.856	0.008
xiii)	13	0.852	0.004
xiv)	14	0.856	0.004
xv)	15	0.857	0.001
xvi)	16	0.856	0.001
xvii)	17	0.861	0.005
xviii)	18	0.855	0.006
xix)	19	0.856	0.001
xx)	20	0.856	0.000
xxi)	21	0.858	0.002
xxii)	22	0.865	0.007
	Total	18.899	0.064

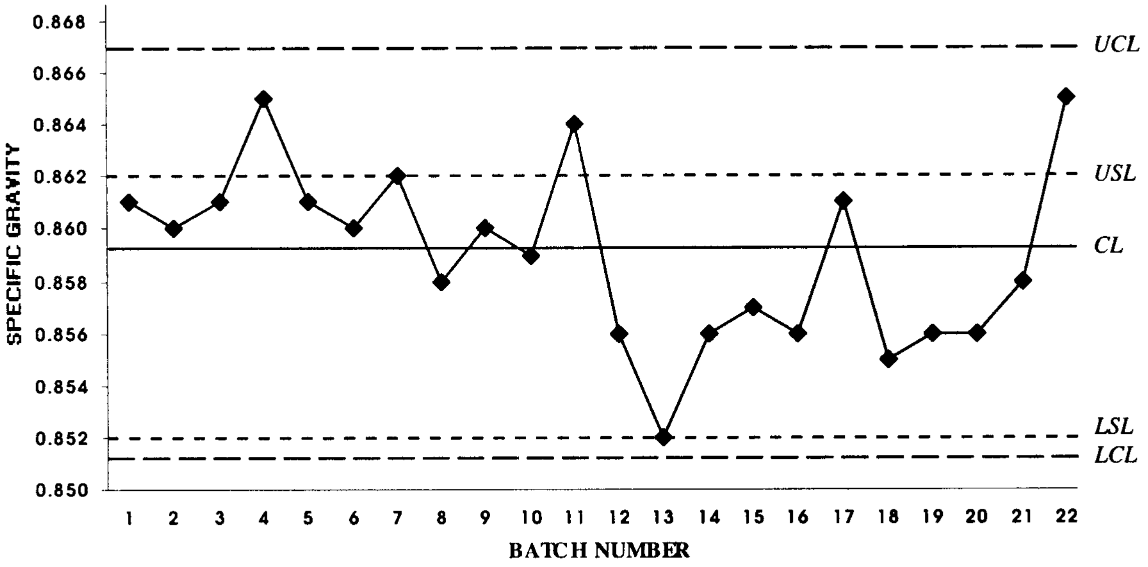

FIG. 3 CHART FOR INDIVIDUALS

Table 4 Bag Weights

(*Clause* 11.4.1)

Product	: Ordinary Portland Cement, 33 Grade IS No.: 269	Characteristic	: Weighment (kg)
Sample size	: 5 bags each of 50 kg (nominal mass)	Frequency	: Every hour from a nozzle of a machine

Hour (1)	Test Results (2)	(3)	(4)	(5)	(6)	Median (7)	Range (8)
1	50.00	50.20	50.00	50.40	50.60	50.20	0.60
2	50.20	50.00	50.40	50.40	50.40	50.40	0.40
3	50.60	50.40	50.20	50.20	50.60	50.40	0.40
4	50.80	50.20	49.50	50.20	49.80	50.20	1.30
5	50.40	49.80	49.20	50.20	50.40	50.20	1.20
6	49.80	50.40	50.60	50.20	50.20	50.60	0.60
7	50.40	50.20	50.80	50.80	50.60	50.20	0.60
8	50.20	50.20	50.40	50.20	50.80	50.60	0.60
10	50.20	50.40	50.20	49.80	50.00	50.20	0.60
11	50.40	50.40	50.60	49.50	50.40	50.20	0.60
12	50.40	50.40	49.80	50.20	50.40	50.20	0.60
13	50.60	50.20	50.40	50.80	50.20	50.40	1.10
14	50.60	49.80	49.20	50.40	50.80	50.40	0.60
15	49.50	50.20	50.20	50.50	50.80	50.40	0.60
16	50.60	50.20	50.40	49.80	50.40	50.60	1.60
17	50.80	50.20	51.00	50.40	50.20	50.20	1.30
18	52.10	52.30	52.50	51.50	51.50	52.10[1]	1.00
19	51.00	52.30	52.10	52.50	52.50	52.30[1]	1.50
20	50.40	52.40	52.30	52.40	51.50	52.30	2.00[1]
21	50.80	50.20	50.80	50.40	50.00	50.40	0.80
22	50.80	50.40	50.20	50.60	50.50	50.50	0.60
23	50.50	50.00	50.80	50.60	50.80	50.60	0.80
24	50.40	50.40	49.80	50.20	50.40	50.40	0.60
25	50.40	50.40	49.80	50.20	50.40	50.40	1.00
					Total	1264.60	22.20

[1] Indicates data dropped for homogenization.

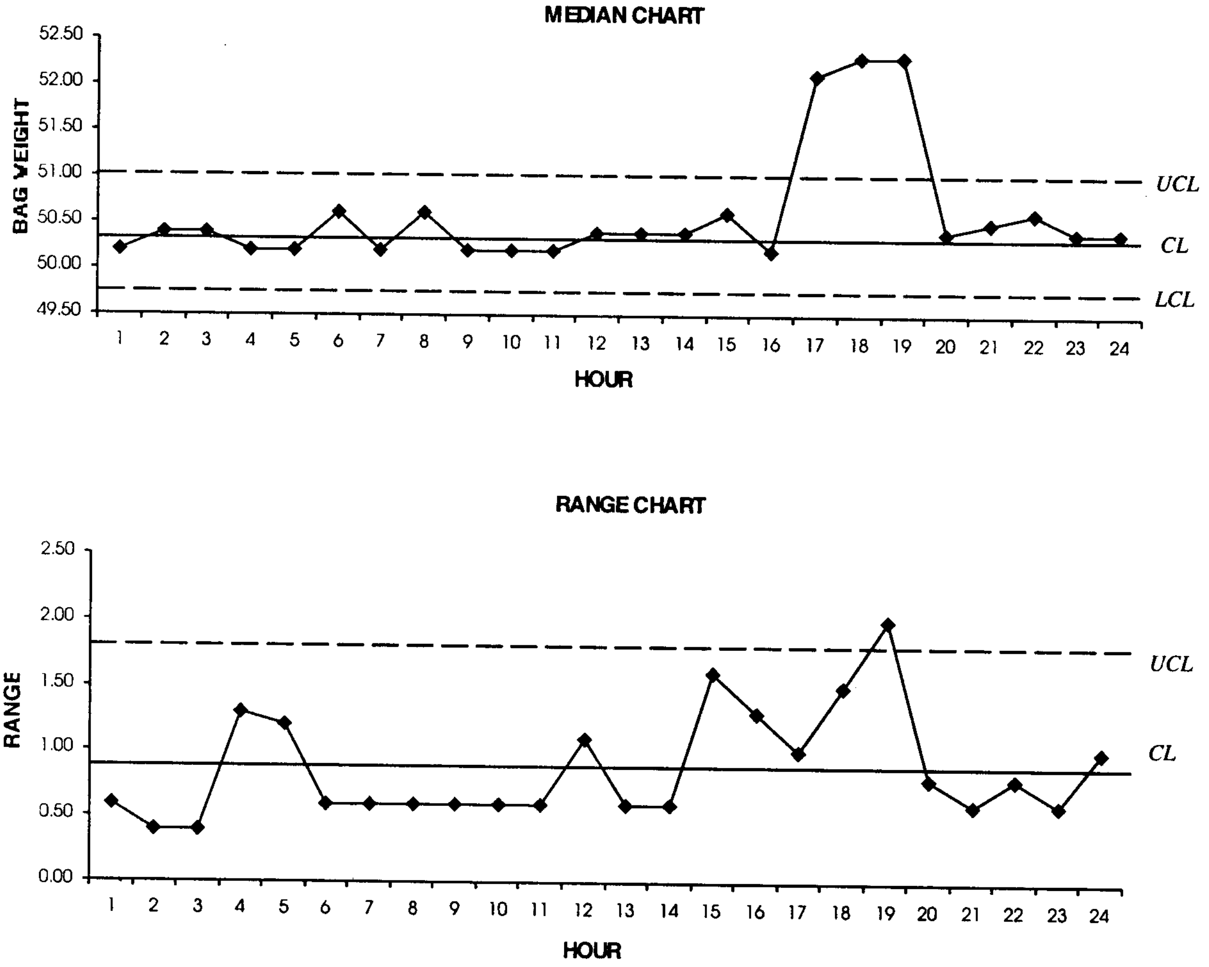

FIG. 4 CONTROL CHART FOR MEDIAN AND RANGE

ANNEX A

(*Clauses* 6.2.2.1, 6.2.2.2, 6.2.3.1, 6.2.3.2, 6.2.3.3, 6.3.1.1, 6.3.1.2, 6.3.2.1, 6.3.2.2, 6.3.2.3, 6.4.2, 6.4.3.1, 6.4.3.2, 6.4.4.1, 6.4.4.2, 6.4.4.3, 9 *and* 11.1.4)

FACTORS FOR COMPUTING CONTROL LIMITS

	Using Standard Values of μ and σ									Using s			Using R				
	Average Chart	Median Chart	Mid-Range Chart	Standard Deviation			Range Chart			Average Chart	Standard Deviation		Average Chart	Median Chart	Mid-Range Chart	Range Chart	
CL	μ	μ	μ	$c_2\sigma$			$d_2\sigma$			$\bar{x}$	$\bar{s}$		$\bar{\bar{x}}$	$\bar{M}_e$	$\bar{M}$	$\bar{R}$	
LCL	$\mu - A\sigma$	$\mu - F\sigma$	$\mu - G\sigma$	$B_1\sigma$			$D_1\sigma$			$\bar{x} - A_1\bar{s}$	$B_3\bar{s}$		$\bar{\bar{x}} - A_2\bar{R}$	$\bar{M}_e - F_2\bar{R}$	$\bar{M} - G_2\bar{R}$	$D_3\bar{R}$	
UCL	$\mu + A\sigma$	$\mu + F\sigma$	$\mu + G\sigma$	$B_2\sigma$			$D_2\sigma$			$\bar{x} + A_1\bar{s}$	$B_4\bar{s}$		$\bar{\bar{x}} + A_2\bar{R}$	$\bar{M}_e + F_2\bar{R}$	$\bar{M} + G_2\bar{R}$	$D_4\bar{R}$	
No. of Observations in Sample	A	F	G	c_2	B_1	B_2	d_2	D_1	D_2	A_1	B_3	B_4	A_2	F_2	G_2	D_3	D_4
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
2	2.121	2.121	2.121	0.564 2	0	1.843	1.128	0	3.686	3.760	0	3.267	1.880	1.880	1.880	0	3.267
3	1.732	2.009	1.805	0.723 6	0	1.858	1.693	0	4.358	2.394	0	2.568	1.023	1.187	1.067	0	2.575
4	1.500	1.638	1.638	0.797 9	0	1.808	2.059	0	4.698	1.880	0	2.266	0.729	0.796	0.796	0	2.282
5	1.342	1.607	1.6532	0.840 7	0	1.756	2.326	0	4.918	1.596	0	2.089	0.577	0.691	0.659	0	2.115
6	1.225	1.300	1.458	0.868 6	0.226	1.711	2.534	0	5.078	1.410	0.030	1.970	0.483	0.549	0.575	0	2.004
7	1.134			0.888 2	0.105	1.672	2.704	0.205	5.203	1.277	0.118	1.882					
8	1.061			0.902 7	0.167	1.638	2.847	0.387	5.307	1.175	0.185	1.815					
9	1.000			0.913 9	0.219	1.609	2.970	0.546	5.394	1.094	0.239	1.761					
10	0.949			0.922 7	0.262	1.584	3.078	0.687	5.469	1.028	0.284	1.716					
11	0.905			0.930 0	0.299	1.561	3.173	0.812	5.534	0.973	0.321	1.679					
12	0.866			0.935 9	0.331	1.541	3.258	0.924	5.592	0.925	0.354	1.646					
13	0.832			0.941 0	0.359	1.523	3.336	1.026	5.646	0.884	0.382	1.618					
14	0.802			0.945 3	0.384	1.507	3.407	1.121	5.693	0.848	0.406	1.594					
15	0.775			0.949 0	0.406	1.492	3.472	1.207	5.737	0.916	0.428	1.572					
16	0.750			0.952 3	0.427	1.478	3.532	1.285	5.779	0.788	0.448	1.552					
17	0.728			0.955 1	0.445	1.465	3.588	1.359	5.817	0.762	0.466	1.534					
18	0.707			0.957 6	0.461	1.454	3.640	1.426	5.854	0.738	0.482	1.518					
19	0.688			0.959 9	0.477	1.443	3.689	1.490	5.888	0.717	0.497	1.503					
20	0.671			0.961 9	0.491	1.433	3.735	1.548	5.922	0.697	0.510	1.490					
21	0.655			0.963 8	0.504	1.424	3.778	1.606	5.950	0.679	0.523	1.477					
22	0.640			0.965 5	0.516	1.415	3.819	1.659	5.979	0.662	0.534	1.466					
23	0.626			0.967 0	0.527	1.407	3.858	1.710	6.006	0.647	0.545	1.455					
24	0.612			0.968 4	0.538	1.399	3.895	1.759	6.031	0.632	0.555	1.455					
25	0.600			0.969 6	0.548	1.392	3.931	1.804	6.059	0.619	0.565	1.435					

NOTES

1 Since the efficiency of median/mid-range as an estimate of central tendency declines as sample size increases, they are not recommended for large sample sizes. For this reason, the entries in col 3 and 4 are restricted to sample up to 6 only.

2 Since range is not recommended for large sample size, the entries in col 14 to 18 are restricted to samples up to 6 only.

ANNEX B

(*Clause* 6.5.2)

CONTROL CHART DATA SHEET (VARIABLES)

Sheet No.

Product:
Characteristic:
Unit of measurement:
Nominal value:
Tolerance:

Sample size:
Frequency:
Date:
Operator:
Inspector:

Production order No.:
Workshop :
Machine No.:

Sl No.	*Date*	*Time*	*Individual Measurement*						*Total*	*Mean/ Median/Mid-Range*	*Range*	*Remarks*
			1	2	3	4	5	6				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
								Total				
								Mean				

ANNEX C

(*Clause* 10.5)

FACTORS FOR COMPUTING MODIFIED CONTROL LIMITS

No. of Observations in the Sample *n*	V_1 *Using Known Standard Deviation*	V_2 *Using Average Range* $\overline{R}$	*No. of Observations in the Sample* *n*	V_1 *Using Known Standard Deviation*	V_2 *Using Average Range* $\overline{R}$
(1)	(2)	(3)	(1)	(2)	(3)
2	0.879	0.779	14	2.198	0.645
3	1.268	0.749	15	2.225	0.641
4	1.500	0.729			
5	1.658	0.713	16	2.250	0.637
			17	2.272	0.633
6	1.775	0.700	18	2.293	0.630
7	1.866	0.690	19	2.312	0.627
8	1.939	0.681	20	2.329	0.624
9	2.000	0.673			
10	2.051	0.666	21	2.345	0.621
			22	2.360	0.618
11	2.095	0.660	23	0.374	0.615
12	2.134	0.655	24	2.388	0.613
13	1.168	0.650	25	2.400	0.611

ANNEX D

(*Foreword*)

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(*Continued on page* 23)

(*Continued from page* 22)

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